Quiz 9: 16.2, 16.3

Show all work clearly. Name any theorems you use. (You may only use theorems from these sections)

(1) Given the vector field $\vec{F}(x,y) = <4x+5y, 5x-y>$ and the path C from (0,0) to (1,1) along the curve.

$$\begin{cases} x = t \\ y = \sin\left(\frac{\pi t}{2}\right) \end{cases}$$

a) Find the potential function f(x,y) such that $\vec{F} = \vec{\nabla} f(x,y)$.

$$f(x,y) = 2x^2 + 5xy - \frac{1}{2}y^2 + C$$

Check $\vec{\nabla} f = \vec{F}$

b) Find $\int_C \vec{F} \bullet d\vec{r}$ using two different methods. Explain

a) Fundamental Theorem

$$\int_C \vec{F} \bullet d\vec{r} = f(1,1) - f(0,0) = 2+5-\frac{1}{2} = \frac{13}{2}$$

Can also do directly, but why?

$$\vec{F} = \left< 4t + 5\sin\frac{\pi t}{2},\ 5t - \sin\frac{\pi t}{2} \right>$$

$$\vec{r}\,' = \left< 1,\ \frac{\pi}{2}\cos\frac{\pi t}{2} \right>$$

$$\int_C \vec{F} \bullet d\vec{r} = \int_0^1 (\vec{F} \cdot \vec{r}\,')\,dt$$

$$= \int_0^1 4t + 5\sin\frac{\pi t}{2} + \underbrace{\frac{5\pi}{2} t \cos\frac{\pi t}{2}}_{\text{parts}} - \underbrace{\frac{\pi}{2}\sin\frac{\pi t}{2}\cos\frac{\pi t}{2}}_{u = \sin\frac{\pi t}{2}}\,dt$$

$\vdots$

$$= \frac{13}{2}$$

b) Since $\vec{F}$ is conservative, we can use a simpler path from (0,0) to (1,1)

C_1: Line segment

$\vec{r} = <t,t>$ $0 \le t \le 1$

$\vec{F} = <4t+5t, 5t-t> = <9t, 4t>$

$\vec{r}\,' = <1,1>$

$\vec{F} \cdot \vec{r}\,' = 13t$

$$\int_C \vec{F} \bullet d\vec{r} = \int_{C_1} \vec{F} \bullet d\vec{r} = \int_0^1 13t\,dt = \frac{13}{2}$$